Int. J. Heat Mass Transfer. Vol. 19, pp. 157-164. Pergamon Press 1976. Printed in Great Britain

THE EFFECT OF AXIAL RADIATION ON THE CARTESIAN GRAETZ PROBLEM

GEORGE C. DAVIS*
General Motors Research Laboratories, Warren, MI 48090, U.S.A.

(*Received* 26 *February* 1974 *and in revised form* 4 *April* 1975)

Abstract—The problem of heat transfer for laminar flow between two infinite horizontal parallel plates with a coordinate system chosen at the middle plane such that the walls at $y = \pm 1$ and $x < 0$ are held at a constant temperature T_0 while the walls at $y = \pm 1$ and $x > 0$ are held at a different constant temperature T_w, is formulated to take into account the effect of radiation on the incoming fluid. Temperature distributions for the regions $x < 0$ and $x > 0$ are obtained for mirror boundaries and black boundaries. This has been achieved by obtaining solutions to the energy equation, including radiative terms, coupled with the radiative transfer equation and by imposing continuity conditions on the temperature and the radiative internal energy and their derivatives at the junction $x = 0$. A parameter survey is made to study the thermal effects of optical thickness, nongrayness and Planck number (ratio of conduction to black body radiation) on the nonscattering, absorbing, and emitting fluid. For mirror boundaries, it is shown that axial radiation is negligible, even at low Peclet numbers, if radiation is appreciable. In the case of black boundaries, axial radiation is negligible only when radiation effects are small and the Peclet number is large.

NOMENCLATURE

- c, specific heat;
- E, emissive power;
- Gz, Graetz number;
- I, intensity;
- K, thermal conductivity;
- L, half distance between plates;
- Nu, Nusselt number;
- Pe, Peclet number, $= RePr$;
- Pr, Prandtl number, $= \mu c/K$;
- q, heat flux;
- Re, Reynolds number, $= \dfrac{UL}{\nu}$;
- T, temperature;
- u, velocity;
- U, maximum velocity;
- x_1, coordinate in horizontal direction;
- y_1, coordinate in vertical direction;
- z_1, coordinate perpendicular to paper;
- x, dimensionless coordinate, $= x_1/L$;
- y, dimensionless coordinate, $= y_1/L$;
- $\mathscr{P}_1$, Planck number, Domain I $= K/L/4\sigma T_0^3$;
- $\mathscr{P}_2$, Planck number, Domain II $= K/L/4\sigma T_w^3$.

Greek symbols

- α_P, Planck mean;
- α_R, Rosseland mean;
- α_m, mean absorptivity, $= (\alpha_R \alpha_P)^{1/2}$;
- σ, Stefan–Boltzmann constant;
- σ_n, eigenvalues;
- ε, emissivity;
- ξ, radiative internal energy;
- λ, $1/4(1/\varepsilon - 1/2)$;
- λ_n, $(2n-1)\pi/2$;
- η, nongrayness parameter, $= (\alpha_P/\alpha_R)^{1/2}$;
- τ, optical thickness, $= \alpha_m L$;
- χ, ratio of nongrayness to Planck number, $= \eta/\mathscr{P}$;
- μ, absolute viscosity;
- ν, kinematic viscosity;
- ϕ_1, dimensionless radiative internal energy Domain I $= \xi_1/4\sigma T_0^4 - 1$;
- ϕ_2, dimensionless radiative internal energy Domain II $= \xi_2/4\sigma T_w^4 - 1$;
- θ_1, dimensionless temperature Domain I $= T_1/T_0 - 1$;
- θ_2, dimensionless temperature Domain II $= T_2/T_w - 1$;
- θ^*, dimensionless temperature $(T_i - T_w)/(T_0 - T_w)$, $i = 1, 2$.

Subscripts

- b, black;
- 0, wall ($x < 0$);
- p, Planck;
- r, Rosseland;
- w, wall ($x > 0$);
- 1, Domain I ($x < 0$);
- 2, Domain II ($x > 0$).

Superscripts

- R, radiation.

*Work was done under the supervision of Professor V. S. Arpaci of the Mechanical Engineering Department, University of Michigan, Ann Arbor, Michigan.

INTRODUCTION

DURING the past decade, considerable attention was given to the problem of radiation effects on laminar flow inside tubes and between parallel plates. In reviewing the literature, the intention was not to give a detail report on what has been done to date but rather to give attention only to those contributions that are directly related to the present work.

Viskanta [1] was among the first to consider the effect of radiation on the well known Nusselt–Graetz problem. Neglecting entrance effects due to both radiation and conduction and considering Cartesian geometry, Viskanta used an iterative procedure to solve the fully developed case. He considered the gray gas with black boundaries and the entire range of optical thickness. Assuming slug flow, *Einstein* [2, 3] extended the Graetz problem to include radiation for a gray gas and black boundaries for both parallel plates and tubes. He assumed the inlet gas temperature to be uniform and the gas absorptivity to be independent of temperature. To account for variation of absorption and emission in the gas, Einstein used distributed energy sources. In each study the integro-differential energy equation was replaced by a system of algebraic equations by using a zonal method given by Hottel and Cohen [4].

Assuming uniform inlet temperatures, Desoto [5] studied the radiative Graetz problem considering a nongray, nonisothermal gas with black boundaries including entrance effects due to radiation and convection. His results included temperature distributions compared with the Graetz problem with no radiation. Desoto found that for CO_2 at about 2500°F radiation caused a drastic effect on the gas temperature. Also, the local heat flux at the wall of the tube was given. His solution technique was iterative in nature. First he would estimate the temperature distribution, calculate the heat flux, and then compute the temperature from the energy using finite difference approximations. A new radiative flux distribution was then computed along with a new temperature distribution. This procedure was iterated until an acceptable result was obtained.

Using Laplace transforms and a numerical iterative technique, Greif and McEligot [6] investigated the effect of one-dimensional thin gas radiation on the Cartesian Graetz problem with black boundaries. They were able to show the effect of the conduction to radiation parameter, N†, on the local heat transfer at the wall. The transverse temperature distribution was given also as a function of this parameter. They found that for relatively small values of N, the temperature gradient at the wall approached zero which caused the local heat flux at the wall, due to conduction, to approach zero.

The foregoing literature survey reveals that the effect of radiation on the general Graetz problem, parallel plates and tubes, has been treated in several aspects. However, none of the above investigators considered both q_x^R, q_y^R and radiant heat penetration out of the usual defined single domain. The aim of the present investigation is to consider both the terms q_x^R and q_y^R along with a two domain problem. These effects together with mirror and black boundaries, Planck number (ratio of conduction to black body radiation)‡ and the entire range of optical thickness are examined using the approximate differential technique which is considered next.

†This parameter is defined by Sparrow and Cess [7] p. 258.

FORMULATION

Considering a radiating fluid flowing in plane poiseuille flow from left to right between infinite parallel plates as shown in Fig. 1. These plates have a temperature T_0 to the left of the origin and a temperature T_W to the right of it.

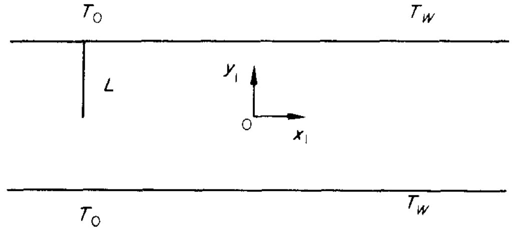

FIG. 1. Laminar flow between parallel plates.

The major consideration in formulating this problem is to investigate the contribution of axial heat transfer by radiation. In order to investigate axial radiation it is necessary to consider a two domain problem and include the term q_x^R in both the energy and transfer equations. This is accomplished by formulating the problem for both domains and using the natural boundary conditions of equality of temperature and temperature gradient at the interface.

It is a well known fact that an exact treatment of radiative transfer in a fluid leads to a formulation in terms of integro-differential equations. Approximate theories have been developed which permit a formulation including only differential equations. As explained by Sparrow and Cess [7], there are a number of ways these differential equations may be obtained. One way involves satisfying certain moments of the equations of transfer, while another proceeds by expanding the intensity I in a series of spherical harmonics. Using the moment method, Arpaci and Gozum [8] obtained a differential form of the transfer equation including both the terms q_y^R and q_x^R. They also formulated boundary conditions that included the color of boundaries and a weighted non-grayness of the gas. In the present work, this formulation is employed.

The usual assumptions of constant fluid properties, negligible viscous dissipation and negligible radiative contributions to momentum are employed. The differential approximation to radiation employing the Planck and Rosseland means in the form of $(\alpha_P \alpha_R)^{1/2}$ satisfies both the thin and thick gas limits and the trendwise behavior between these extremes is qualitatively accurate.§ However, this approximation consistently predicts lower gas temperatures between these limits. Therefore, care must be taken in applying this approximation to gases such as CO, CO_2, H_2O, SO_2,

‡This parameter is discussed later in the text.

§For further discussion, specific examples and details see references [8–14].

the hydrocarbons and the high temperature oxygen, nitrogen, and hydrogen plasmas.

With the above assumptions and relating black body radiation to temperature ($E_b = \sigma T^4$) the formulation of the problem in dimensionless form may be stated as:

$$Pe(1-y^2)\frac{\partial\theta_i}{\partial x} = \frac{\partial^2\theta_i}{\partial x^2} + \frac{\partial^2\theta_i}{\partial x^2} + \chi_i\tau(\phi_i - 4\theta_i)$$
$$\frac{\partial^2\phi_i}{\partial x^2} + \frac{\partial^2\phi_i}{\partial y^2} = 3\tau^2(\phi_i - 4\theta_i)\dagger. \quad (1)$$

The boundary conditions are:

$$\theta_i(\pm\infty, y) = 0, \qquad \phi_i(\pm\infty, y) = 0$$
$$\frac{\partial\theta_i}{\partial y}(x, 0) = 0, \qquad \frac{\partial\phi_i}{\partial y}(x, 0) = 0 \quad (2)$$
$$\theta_i(x, 1) = 0, \qquad \phi_i(x, 1) + \frac{\eta}{3\lambda\tau}\frac{\partial\phi_i}{\partial y}(x, 1) = 0.$$

The interface conditions are:

$$\theta_1(0, y) = \frac{T_w}{T_0}\theta_2(0, y) + \frac{T_w}{T_0} - 1,$$
$$\phi_1(0, y) = \frac{T_w^4}{T_0^4}\phi_2(0, y) + \frac{T_w^4}{T_0^4} - 1$$
$$\frac{\partial\theta_i}{\partial x}(0, y) = \frac{T_w}{T_0}\frac{\partial\theta_2}{\partial x}(0, y), \quad \frac{\partial\phi_1}{\partial x}(0, y) = \frac{T_w^4}{T_0^4}\frac{\partial\phi_2}{\partial x}(0, y).$$

Since we are interested in high temperature levels but not large temperature differences, the last term of equation set (1) may be linearized.

The results for the mirror and black surfaces are readily obtainable from the foregoing equations by considering the limits $\lambda = 0$ and $\lambda = 1/2$, respectively.

SOLUTION

Reduction to a differential eigenvalue problem

In general, an exact solution of the present formulation is not possible. However, an approximate scheme, the Galerkin method, has proven to be successful in earlier works, and is employed here for the purpose of the solution. The method has been discussed by Finlayson [16] and Finlayson and Scriven [17]. The variables θ and ϕ are expanded in complete sets of orthogonal functions which satisfy the boundary conditions. The coefficients of these functions are chosen by forcing the errors resulting from the substitution of these functions into the original differential equations to be orthogonal to the trial functions in the domain of interest.

Due to the physical symmetry of the problem, the solution is composed of even functions only. In view of this, the boundary conditions, equation set (2), suggest that,

$$\theta = \sum_{n=1}^{N} A_{nj}(x)\cos\lambda_n y, \text{ where}$$
$$\lambda_n = (2n-1)\pi/2, \quad (3)$$

and

$$\phi = \sum_{n=1}^{N} B_{nj}(x)\cos\mu_n y, \quad (4)$$

†For a detail development of this equation see reference [15].

is a proper orthogonal set to represent the temperature θ and radiative internal energy ϕ. The μ_n's are given by the roots of the transcendental equation,

$$\tan\mu = 3\lambda\tau/\eta\mu. \quad (5)$$

For the special case of mirror boundaries, $\mu_n = n\pi$.

Substituting equations (3) and (4) into the set (1) and orthogonalizing with respect to $\cos\lambda_n y$ and $\cos\mu_n y$ over the interval $0 \leqslant y \leqslant 1$, we obtain an infinite set of simultaneous second order linear ordinary differential equations with $A_{nj}(x)$ and $B_{nj}(x)$ as the only unknowns for both the regions $x < 0$ and $x > 0$:

$$-PeS_{nm}\frac{dA_{nj}}{dx} - H_{nm}A_{nj} + \frac{\delta_{nm}d^2A_{nj}}{2dx^2} + \frac{\eta\tau}{\mathscr{P}_i}Q_{nm}B_{nj} = 0$$
$$F_{nm}\frac{d^2B_{nj}}{dx^2} - G_{nm}B_{nj} + 12\tau^2 Q_{nm}A_{nj} = 0. \quad (6)$$

The above system may be reduced to a set of $4N$ first order differential equations with $4N$ unknowns. To obtain the first order set, we define:

$$C_{nj} = \frac{dA_{nj}}{dx} \quad \text{and} \quad D_{nj} = \frac{dB_{nj}}{dx}.$$

The new system may be written as:

$$-PeS_{nm}\frac{dA_{nj}}{dx} - H_{nm}A_{nj} + \frac{\delta_{nm}}{2}\frac{dC_{nj}}{dx} + \frac{\eta\tau}{\mathscr{P}_i}Q_{nm}B_{nj} = 0$$
$$F_{nm}\frac{dD_{nj}}{dx} - G_{nm}B_{nj} + 12\tau^2 Q_{nm}A_{nj} = 0$$
$$\delta_{nm}\frac{dA_{nj}}{dx} - \delta_{nm}C_{nj} = 0 \quad (7)$$
$$\delta_{nm}\frac{dB_{nj}}{dx} - \delta_{nm}D_{nj} = 0.$$

Or, the general form may be expressed as:

$$P_{nm}\frac{dX_{mj}}{dX} = E_{nm}X_{mj}. \quad (8)$$

The explicit forms of the matrices S_{nm}, H_{nm}, Q_{nm}, F_{nm}, G_{nm}, P_{nm}, E_{nm} and X_{mj} are given in [15].

The differential eigenvalue problem is solved by finding the latent roots of the matrix $P_{nm}^{-1}E_{nm}$. Eigenvalues for a given value of N are obtained by transforming the matrix $P_{nm}^{-1}E_{nm}$ to upper almost triangular form (Hessenberg form) and then employing the QR algorithm [18]. Subroutines for both these methods exist in the scientific subroutine package (IBM) system. The accuracy of the eigenvalue subroutine is tested by computing the difference between the trace (the sum of the diagonals of $P_{nm}^{-1}E_{nm}$) and the sum of the eigenvalues. In all cases, this difference was found to be less than 0·01 per cent. When an N-term approximation is used in the expansion (8), $P_{nm}^{-1}E_{nm}$ has $4N$ real eigenvalues. Furthermore, $P_{nm}^{-1}E_{nm}$ has an infinite number of positive and negative roots. The negative eigenvalues $\sigma_{-j}(j = 1, 2, 3, \ldots, 2N)$ are admissible in the region $x > 0$ and the positive eigenvalues $\sigma_{+j}(j = 1, 2, 3, \ldots, 2N)$ in the region $x < 0$, because of the boundary conditions on θ and ϕ at $x = \pm\infty$. Eigenvalues for each domain are obtained from equation (7) provided $\mathscr{P}_1$ is replaced by $\mathscr{P}_2$.

After the eigenvalues are determined, the constants A_{nj} and B_{nj} are solved for the governing equations and interface conditions. Note that the solution in the x-direction is of exponential form; therefore we have

$$A_{nj}(x) = a_{nj}\exp(\sigma_j x) \tag{9}$$

and

$$B_{nj}(x) = b_{nj}\exp(\sigma_j x). \tag{10}$$

Substituting these expressions into the set (6) we get:

$$\begin{aligned} -PeS_{nm}\sigma_j a_{nj} - H_{nm}a_{nj} + \frac{\delta_{nm}}{2}\sigma_j^2 a_{nj} + \frac{\eta\tau}{\mathscr{P}_i}Q_{nm}b_{nj} &= 0 \\ F_{nm}\sigma_j^2 b_{nj} - G_{nm}b_{nj} + 12\tau^2 Q_{nm}a_{nj} &= 0 \end{aligned} \tag{11}$$

or,

$$\begin{aligned} a_{nj}AA_{nm} + b_{nj}BB_{nm} &= 0 \\ a_{nj}CC_{nm} + b_{nj}DD_{nm} &= 0 \end{aligned} \tag{12a}$$

where AA_{nm}, BB_{nm}, CC_{nm} and DD_{nm} are given in [15].

The system (12a) forms a set of homogeneous algebraic equations which give a unique set of a_{nj} and b_{nj} for every eigenvalue σ_j, with a non-trivial solution existing only if the determinant of the coefficient matrix vanishes. Therefore, the constants a_{nj} and b_{nj} are determinable only in terms of a_{1j}. Hence, the $4N$ eigenvalues give $8N^2$ constants, a_{nj} and b_{nj} to be completely determined using the conditions given at the interface $x = 0$. The eigenvalues σ_{-j} give the constants, a_{-nj} and b_{-nj}, for the region $x > 0$ and the eigenvalues σ_{+j} give the constants a_{+nj} and b_{+nj} for the region $x < 0$.

The connecting procedure at the interface of the two domains is carried out by using the assumed expressions for θ and ϕ. The first interface condition is the equality of temperature and is given by

$$\theta_1(0, y) = \theta_2(0, y)\frac{T_w}{T_0} + \frac{T_w}{T_0} - 1.$$

Substituting the expressions for θ_1 and θ_2, equation (3), gives:

$$\begin{aligned} \frac{T_w - T_0}{T_w} = &-\sum_{j=1}^{2N}\sum_{n=1}^{N} A_j^* a_{nj}\cos\lambda_n y \\ &+ \frac{T_0}{T_w}\sum_{j=2n+1}^{4N}\sum_{n=1}^{N} A_j^* a_{nj}\cos\lambda_n y. \end{aligned} \tag{13}$$

To simplify equation (13), we multiply by $\cos\lambda_m y$ and integrate over the interval $0 \leqslant y \leqslant 1$. This orthogonalization process gives:

$$2\left(\frac{T_0}{T_w} - 1\right)\frac{(-1)^n}{\lambda_n} = -\sum_{n=1}^{2N} A_j^* a_{nj} + \frac{T_0}{T_w}\sum_{j=2n+1}^{4N} A_j^* a_{nj} \tag{14}$$

where,

$$n = 1, 2, 3, \ldots, N.$$

Likewise, the temperature gradient,

$$\frac{\partial\theta_1}{\partial x}(0, y) = \frac{T_w}{T_0}\cdot\frac{\partial\theta_2}{\partial x}(0, y),$$

at the interface gives:

$$0 = -\sum_{j=1}^{2N}\sigma_j A_j^* a_{nj} + \frac{T_0}{T_w}\sum_{j=2n+1}^{4N}\sigma_j A_j^* a_{nj}. \tag{15}$$

Also, equating the radiative internal energy,

$$\phi_1(0, y) = \frac{T_w^4}{T_0^4}\cdot\phi_2(0, y) + \frac{T_w^4}{T_0^4} - 1$$

and its derivative,

$$\frac{\partial\phi_1}{\partial x}(0, y) = \frac{T_w^4}{T_0^4}\frac{\partial\phi_2}{\partial x}(0, y)$$

at the interface give:

$$\begin{aligned} &4\left(1 - \frac{T_0}{T_w}\right)\frac{4\mu_n\sin\mu_n}{2\mu_n^2 + \sin 2\mu_n} \\ &\quad = -\sum_{j=1}^{2N} A_j^* b_{nj} + \left(\frac{4T_0}{T_w} - 3\right)\sum_{j=2n+1}^{4N} A_j^* b_{nj} \end{aligned} \tag{16}$$

and,

$$0 = \sum_{j=1}^{2N}\sigma_j A_j^* b_{nj} - \left(\frac{4T_0}{T_w} - 3\right)\sum_{j=2n+1}^{4N}\sigma_j A_j^* b_{nj}. \tag{17}$$

Equations (14)–(17) may be arranged to give,

$$\begin{aligned} -\sum_{j=1}^{2N} A_j^* a_{nj} + \frac{T_0}{T_w}\cdot\sum_{j=2N+1}^{4N} A_j^* a_{nj} &= GG_n \\ -\sum_{j=1}^{2N}\sigma_j A_j^* a_{nj} + \frac{T_0}{T_w}\cdot\sum_{j=2N+1}^{4N}\sigma_j A_j^* a_{nj} &= 0 \\ -\sum_{j=1}^{2N} A_j^* b_{nj} + \left(\frac{4T_0}{T_w} - 3\right)\sum_{j=2N+1}^{4N} A_j^* b_{nj} &= FF_n \\ -\sum_{j=1}^{2N}\sigma_j A_j^* b_{nj} + \left(\frac{4T_0}{T_w} - 3\right)\sum_{j=2N+1}^{4N}\sigma_j A_j^* b_{nj} &= 0 \end{aligned} \tag{18}$$

where FF_n and GG_n are given in [15]. For mirror boundaries, the right side of the third equation is zero. This system gives a set of $4N$ algebraic, linear and nonhomogeneous equations in $4N$ unknowns, $(4N, A_j^*\text{'s})$ where the eigenvalues σ_{-j} give the constants A_{-j}^* for the region $x > 0$ and the eigenvalues σ_{+j} give the constants A_{+j}^* for the region $x < 0$.

Finally, the temperature distributions for $x > 0$ and $x < 0$ are respectively:

$$\theta_2 = \sum_{j=1}^{2N}\sum_{n=1}^{N} e^{\sigma_{-j}x} - (A_j^* a_{nj})\cos\lambda_n y \tag{19}$$

and

$$\theta_1 = \sum_{j=2N+1}^{4N}\sum_{n=1}^{N} e^{\sigma_{+j}x} + (A_j^* a_{nj})\cos\lambda_n y. \tag{20}$$

Likewise the radiative internal energies for $x > 0$ and $x < 0$ are respectively:

$$\phi_2 = \sum_{j=1}^{2N}\sum_{n=1}^{N} e^{\sigma_{-j}x} - (A_j^* b_{nj})\cos\mu_n y \tag{21}$$

and

$$\phi_1 = \sum_{j=2N+1}^{4N}\sum_{n=1}^{N} e^{\sigma_{+j}x} + (A_j^* b_{nj})\cos\mu_n y. \tag{22}$$

The heat transfer at the walls are given by the conduction at the wall and radiative flux at the wall.

$$Nu_i = \frac{\partial T/\partial y|y=1}{\left(1-\frac{T_0}{T_w}\right)} + \frac{\eta}{3\mathscr{P}_i\tau}\frac{\partial\phi/\partial y|y=1}{\left(1-\frac{T_0}{T_w}\right)} \tag{23}$$

for explicit forms of (23) see [15].

DISCUSSION OF RESULTS

Effect of axial radiation with mirror boundaries

The effect of τ on centerline temperature is shown in Fig. 2. The temperature profile for $\tau = 0$ characterizes the non-radiating case, and it agrees with results given by Agrawal [19]. This result shows that as the optical thickness is increased axial radiation diminishes. This is evident from the trend of the centerline temperature gradient at the interface ($1/GZ = 0$) of the two domains as τ is increased. This result is consistent with the so-called "radiation Peclet number" prediction given by Sparrow and Cess [7]. With mirror boundaries, the wall temperature is redundant since they only reflect and do not emit. Therefore, it is meaningless to vary $\mathscr{P}$ since it is independent of τ and other radiant effects except wall temperature. Changing η shows a similar behavior as τ and results are given in [15].

Effect of axial radiation with black boundaries

The effect of $\mathscr{P}$ on centerline temperature is given in Fig. 3. This result shows that as this parameter is decreased, axial radiation increases. Since decreasing $\mathscr{P}$ means increasing both T_0 and T_w, varying this parameter implies a wall effect on axial radiation only. Sparrow and Cess [7] define a conduction to radiation parameter N that includes τ the optical thickness. The relation between this parameter and $\mathscr{P}$ is $N = \mathscr{P}/\tau$. Therefore N exhibits a radiant gas property effect as well as a radiant wall effect, whereas $\mathscr{P}$ only shows a radiant wall effect.

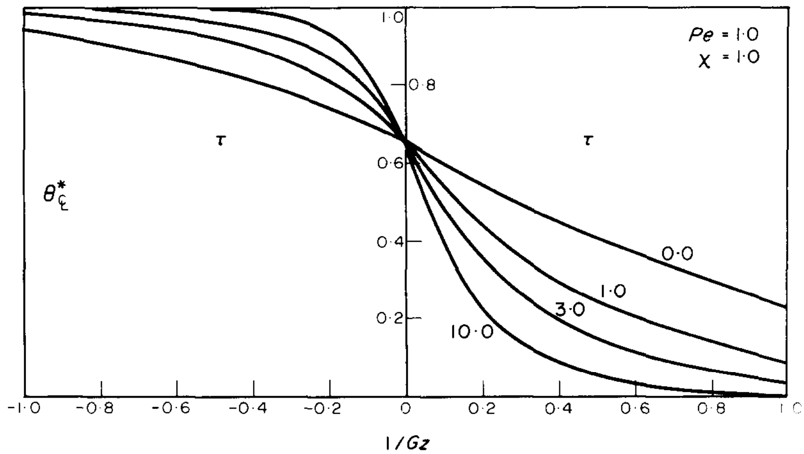

FIG. 2. The effect of optical thickness on centerline temperature for $T_0/T_w = 1{\cdot}2$.

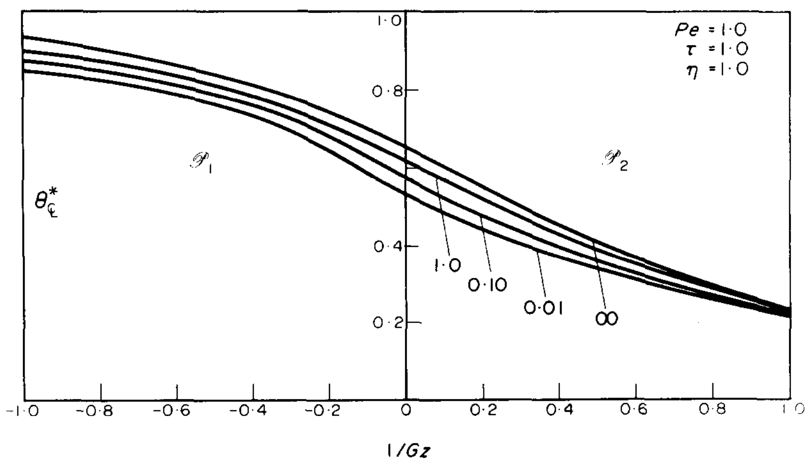

FIG. 3. The effect of Planck number on centerline temperature for $T_0/T_w = 1{\cdot}2$.

The radiant wall effect that is shown in Fig. 3 is not accomplished by increasing the ratio of T_0/T_w but instead by increasing the magnitude of both T_0 and T_w. This is achieved by decreasing either $\mathscr{P}_1$ or $\mathscr{P}_2$ and calculating the other since they are coupled through the relation

$$\mathscr{P}_2 = (T_0/T_w)^3\mathscr{P}_1†.$$

It is well known that for low Peclet numbers on the order of 1, axial conduction is significant. Adding axial radiation, in this case, increases this significance. How-

† The quantity T_0/T_w was fixed at a value of 1·2 for this study.

ever, it is of interest to increase Pe to a value where axial conduction is negligible and investigate the axial radiant effect. Figure 4 shows the effect of $\mathscr{P}$ on centerline temperature for $Pe = 10$ and other conditions the same as before. It is apparent from this result that axial radiation is appreciable since the centerline temperature for $1/Gz < 0$ deviates farther from T_0 as the effect of this parameter is increased.

CONCLUSIONS

It is observed that in the case of mirror boundaries, axial radiation can be neglected even for small Peclet numbers. In the case of black boundaries, axial radiation is negligible only when radiation effects are small. For large Peclet numbers, it is observed that the parameters τ, η and $\mathscr{P}$ control axial radiation. When the parameters τ and η are fixed quantities, the product

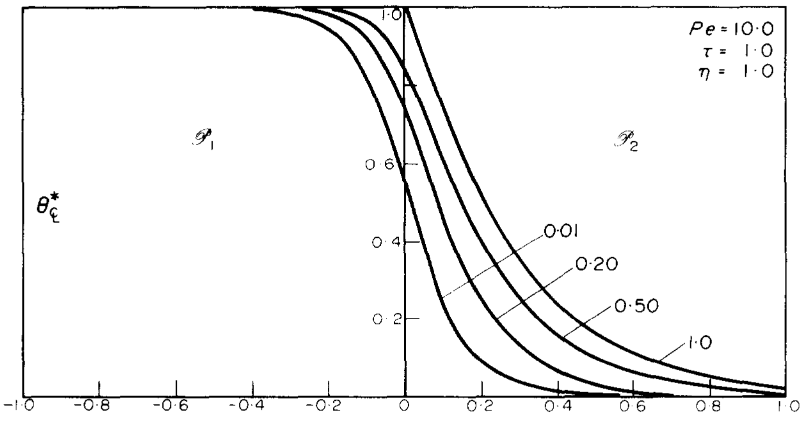

FIG. 4. The effect of Planck number on centerline temperature for $T_0/T_w = 1{\cdot}2$.

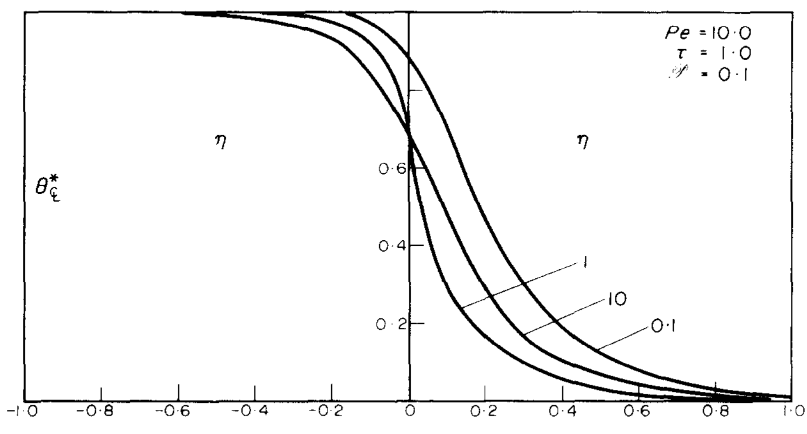

FIG. 5. The effect of non-centerline temperature for $T_0/T_w = 1{\cdot}2$.

Effect of τ

The effect of τ on axial radiation is maximized for τ equals approximately two. This result in general agrees with that given by Viskanta [1]. However, the effect of τ is small when compared with the effect of $\mathscr{P}$. See [15] for exact comparisons.

Effect of η

The effect of η on centerline temperature is shown in Fig. 5. This result indicates that as η is increased axial radiation increases. This is the same trend noted when $\mathscr{P}$ was decreased except for the behavior of θ^* for $1/Gz > 0$. Since $\mathscr{P}$ is fixed, this effect is only due to gas properties and as expected show a similar trend to that produced when Pe is decreased.

$Pe\mathscr{P}$ control axial radiation. An example is the special case of the thin or thick gray gas. For small Peclet numbers, both axial diffusion and radiation are important quantities, and the parameters τ, η, $\mathscr{P}$ and Pe must be specified.

REFERENCES

1. R. Viskanta, Interaction of heat transfer by conduction, convection and radiation in a radiating fluid, *J. Heat Transfer* **C85**, 318–328 (1963).
2. T. H. Einstein, Radiant heat transfer to absorbing gases enclosed between parallel plates with flow and conduction, NASA TR R-154 (1963).
3. T. H. Einstein, Radiant heat transfer to absorbing gases enclosed in a circular pipe with conduction, gas flow and internal heat generation, NASA TR R-156 (1963).

4. H. C. Hottel and E. S. Cohen, Radiant heat exchange in a gas-filled enclosure; allowance for nonuniformity of gas temperature, *Am. Inst. Chem. Engrs Jl* **4**, 3–14 (1958).
5. S. De Soto, Coupled radiation, conduction and convection in entrance region flow, *Int. J. Heat Mass Transfer* **11**, 39–53 (1968).
6. R. Greif and D. M. McEligot, Influence of optically thin radiation on heat transfer in the thermal entrance region of a narrow duct, *J. Heat Transfer* **C93**, 473–475 (1971).
7. E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*. Brooks/Cole, Monterey, CA (1966).
8. V. S. Arpaci and D. Gozum, Thermal stability of radiating fluids—the Benard problem, *Physics Fluids* **5**, 581–588 (1973).
9. S. C. Traugott, Radiation heat flux potential for a nongray gas, *AIAA Jl* **4**, 541–542 (1966).
10. A. C. Cogley, W. G. Vincenti and S. E. Gilles, On the differential approximation for radiative transfer in a nongray gas, *AIAA Jl* **6**, 551–553 (1968).
11. S. E. Gilles, A. C. Cogley and W. G. Vincenti, A substitute-kernal approximation for radiative transfer in a nongray gas near equilibrium with application to radiative acoustics, *Int. J. Heat Mass Transfer* **12**, 445–458 (1968).
12. D. H. Mandell and R. D. Cess, Radiation energy transfer within a hydrogen plasma, *Int. J. Heat Mass Transfer* **13**, 1–12 (1969).
13. D. A. Mandell and R. D. Cess, Radiation transport quantities for a hydrogen plasma, *J. Quantve Spectrosc. & Radiat. Transfer* **9**, 981–994 (1969).
14. G. M. Simmons and J. H. Ferziger, Nongray radiative heat transfer between parallel plates, *Int. J. Heat Mass Transfer* **11**, 1611–1620 (1968).
15. G. C. Davis, The effect of radiation on the cartesian Graetz problem, Ph.D. Thesis, ME Department, University of Michigan, Ann Arbor, Michigan (1973).
16. B. A. Finlayson, *The Method of Weighted Residuals and Variation Principles*. Academic Press, New York (1972).
17. B. A. Finlayson and L. E. Scriven, The method of weighted residuals—a review, *Appl. Mech. Rev.* **19**, 735 (1966).
18. J. H. Wilkinson, *The Algebraic Eigenvalue Problem*. Oxford University Press, Oxford (1965).
19. H. C. Agrawal, Heat transfer in laminar flow between parallel plates at small Peclet numbers, *Appl. Scient. Res.* **A9**, 177 (1960).

L'EFFET DU RAYONNEMENT AXIAL DANS LE PROBLEME DE GRAETZ EN AXES CARTESIENS

Résumé—Le problème du transfert de chaleur en écoulement laminaire entre deux plaques parallèles infinies horizontales a été formulé afin de tenir compte de l'effet du rayonnement sur le fluide entrant, le système de coordonnées est choisi sur le plan médian tel que les parois à $y = \pm 1$ et $x < 0$ soient maintenues à une température constante T_0 tandis que les parois à $y = \pm 1$ et $x > 0$ sont portées à une température constante différente T_w. Les distributions de température dans les régions $x < 0$ et $x > 0$ sont obtenues dans le cas de frontières noires et dans le cas de miroirs. Elles ont été obtenues par une résolution de l'équation d'énergie contenant des termes de rayonnement, couplée à l'équation du transfert par rayonnement et en imposant des conditions de continuité sur la température, sur l'énergie interne de rayonnement et sur leurs dérivées à la jonction $x = 0$. Un examen des paramètres est effectué pour étudier les effets thermiques de l'épaisseur optique, de l'état non gris et du nombre de Planck (rapport de la conduction au rayonnement du corps noir) sur le fluide non diffusif, absorbant et émissif. Dans le cas de frontières constituées de miroirs, on montre que le rayonnement axial est négligeable, même aux faibles nombres de Péclet, si le rayonnement est appréciable. Dans le cas de frontières noires, le rayonnement axial est négligeable seulement lorsque les effets du rayonnement sont faibles et le nombre de Péclet grand.

DER EINFLUSS AXIALER STRAHLUNG AUF DAS RECHTWINKLIGE GRAETZ-PROBLEM

Zusammenfassung—Es wird der Wärmeübergang für laminare Strömung zwischen zwei unendlichen, waagerechten, parallelen Platten untersucht unter Berücksichtigung des Einflusses der Strahlung auf das strömende Fluid. Das Koordinatensystem ist in der Mittelebene gewählt, so daß die Wände bei $y = \pm 1$ und $x < 0$ auf der konstanten Temperatur T_0 sich befinden, während die Wände bei $y = \pm 1$ und $x > 0$ auf einer anderen konstanten Temperatur T_w gehalten sind. Temperaturverteilungen werden für die Bereiche $x < 0$ und $x > 0$ für spiegelnde und schwarze Grenzen erhalten. Dies wurde erreicht durch Lösung der Energiegleichung einschließlich eines Strahlungsterms gekoppelt mit der Gleichung für Temperaturstrahlung und Anwendung der Kontinuitätsgleichungen auf die Temperatur und die durch Strahlung übertragene innere Energie und Berücksichtigung der Ableitungen an der Stelle $x = 0$. Eine Parameterstudie liefert die thermischen Einflüsse der optischen Dicke der Abweichung vom grauen Strahler und der Planck-Zahl (Verhältnis der Leitung zur Strahlung des schwarzen Körpers) auf das nichtstreuende, absorbierende und emittierende Fluid. Auch bei größerem Strahlungsanteil erweist sich für spiegelnde Grenzen die Axialstrahlung als vernachlässigbar bei kleinen Péclet-Zahlen. Für schwarze Berandungen ist die Axialstrahlung nur dann vernachlässigbar, wenn die Strahlungseffekte klein sind und die Péclet-Zahl groß ist.

УЧЕТ ЭФФЕКТА АКСИАЛЬНОГО ИЗЛУЧЕНИЯ В ЗАДАЧЕ ГРЭТЦА В ДЕКАРТОВЫХ КООРДИНАТАХ

Аннотация — Формулируется задача теплопереноса с учетом эффекта излучения на набегающий поток при ламинарном течении между двумя бесконечными горизонтальными параллельными пластинами с координатной системой, выбранной в середине плоскости таким образом, что на стенках при $y = \pm 1$ и $x < 0$ поддерживается постоянная температура T_0, в то время как при $y = \pm 1$ и $x > 0$ поддерживается различная постоянная температура T_w.

Получено распределение температуры при $x<0$ и $x>0$ для зеркальных и черных границ. Для этого, уравнение энергии, включающее члены излучения, решалось совместно с уравнением переноса излучения при условиях неразрывности на температуру и внутреннюю энергию излучения и их производных в точке $x=0$. Приводится обзор параметров по исследованию влияния тепловых эффектов оптической толщины, пропускания и числа Планка (отношение проводимости к излучению черного тела) на нерассеивающую, поглощающую и излучающую жидкость. Если излучение значительно, то показано, что для зеркальных границ аксиальное излучение пренебрежимо мало даже при малых числах Пекле. В случае черных границ, аксиальное излучение пренебрежимо мало только тогда, когда эффекты излучения малы, а число Пекле велико.